Required Cauer network order for modelling of thermal transfer impedance

J.N. Davidson, D.A. Stone and M.P. Foster

Impedance spectroscopy, a technique for analysing electroceramics, is applied to the two-port form of the Cauer network which is commonly used as a thermal circuit analogue. The transfer impedance spectra of Cauer network representations of a single material are presented as Nyquist plots for a range of network orders. Results demonstrate that a model with four or more RC elements is required to accurately model a single material. The effect of taking a measurement at different points within a single material is then reported and four elements are similarly required between heat source and measurement. Existing modelling techniques, which often use only one element per material block, are shown to be especially inaccurate. Using these findings, a design engineer can produce a better thermal model of a complete system by making an informed compromise between model accuracy and complexity.

Introduction: Demand for increased power density in recent years is providing an ever greater impetus for advanced thermal modelling techniques where real-time processing adapts cooling behaviour to the changing thermal state of a system. In this case, thermal modelling of devices in a system is necessary to predict thermal cycling failure which can be reduced by integrated thermal management [1]. Such analyses commonly use electrical circuit analogues, where thermal mass is modelled by a capacitor, C_{th}, and thermal resistance is modelled by a resistor, R_{th}. A current source in such analogues represents heat flux or generation, Q, and voltages represent temperature, T.

Cauer- and Foster-based networks are usually used for modelling. In their one-port form, they are mathematically identical [2]; however, the Cauer network is a closer representation of the physical form of the thermal circuit because each node represents a real temperature. In the Foster network, by contrast, only the leftmost node has physical significance. Additionally, thermal cross-coupling, which is the transfer impedance between two system components, is used to analyse the thermal response of system with multiple heating elements [4]. In this case, a two-port network must be used and since the Foster network does not have an appropriate two-port form, a two-port Cauer network such as the one shown in Fig. 1 is used. In this work, all temperatures are relative to ambient which is represented symbolically by an earth connection.

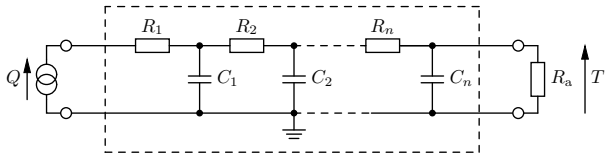

Fig. 1 *Two-port Cauer network with heat source, Q, and ambient connection, R_a*

In this Letter, thermal impedance spectroscopy is used to determine the appropriate order of the Cauer network required in a thermal analysis. Impedance spectroscopy is a technique for analysing the constituent components of ceramics by virtue of their differing electrical time constants [3]. Data is plotted as Nyquist diagrams and characteristics are found by inspection. In recent years this technique has been applied to thermal analysis, particularly for packaging and microelectronics [5, 6]. To illustrate the limitations of present thermal modelling, the transfer impedance between the active region of a TO220-packaged transistor and a thermocouple glued to its tab was measured experimentally using impedance spectroscopy, as shown in Fig. 2. Typical thermal modelling would consider this as a first-order problem and therefore use a single element Cauer network. However, such a model occupies only a single quadrant in the Nyquist diagram whereas the characteristic of the real device occupies multiple quadrants. For even the simplest of thermal arrangements, the present models are inaccurate. An analysis of the network length required for accurate thermal modelling is therefore presented.

Required network order: A device may be modelled as a series of adjoined discrete materials, each with a thermal mass and thermal conductivity. A

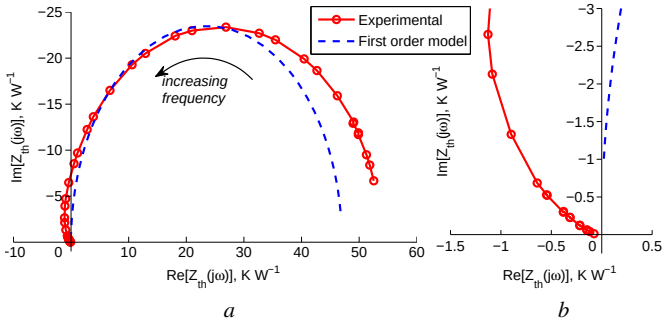

Fig. 2 *Nyquist plot of experimental transfer impedance spectrum for a TO220 device with fitted first order model*

a. Full plot

b. Zoomed in around the origin

Cauer network model can be developed by taking these properties and assigning the corresponding capacitance, C_{th}, and resistance, R_{th}, to each Cauer element. In reality, R_{th} and C_{th} are distributed components; however, to maintain ease of analysis they are lumped together so that their effects can be modelled as discrete components.

In order to build a model of a complete thermal system, the system is first divided into single material blocks each of which is assigned a thermal resistance and capacitance. The complete system model is formed by interconnecting these blocks and inserting heat sources and ambient connections as appropriate. Typically, each single material is modelled as a single $R_{\text{th}}C_{\text{th}}$ element. However, to approximate distributed components, the bulk $R_{\text{th}}C_{\text{th}}$ element for each material block can be divided into n smaller $R_{x(\text{th})}C_{x(\text{th})}$ elements according to equation (1), which results in an n^{th} order Cauer network for each block.

$$R_{x(\text{th})} = \frac{R_{\text{th}}}{n}; \qquad C_{x(\text{th})} = \frac{C_{\text{th}}}{n} \tag{1}$$

While additional elements improve the approximation to a distributed component, they also add to the computational complexity of the model. The order of a Cauer network model is the number of RC elements, n, from which it is formed. To find the optimal order for frequency response modelling, we consider impedance spectra of a single block of material modelled as a Cauer network of varying order. A block defined only by its thermal properties, in this case $R_{\text{th}} = 10$ K/W and $C_{\text{th}} = 0.1$ Ws/K, is modelled as an n^{th} order Cauer network with an ambient connection of resistance $R_{\text{a(th)}} = 10$ K/W to a fixed temperature. We neglect non-linear heat flows, such as convection and radiation, in this work. The thermal impedance spectrum, $Z_{\text{th}}(j\omega)$, is calculated according to equation (2).

$$Z_{\text{th}}(j\omega) = \frac{T}{Q}\angle\phi \tag{2}$$

where j is the imaginary unit, ω is angular frequency, Q is the heat flux into the material, T is the temperature at the boundary with ambient and ϕ is the phase difference between T and Q.

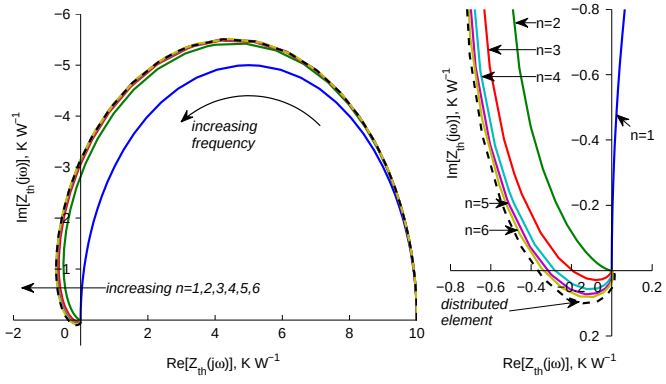

Fig. 3 *Nyquist plots of single material Cauer network approximation of different orders, n*

Fig. 3 shows transfer impedance Nyquist plots of the material for different Cauer network orders, n. As n increases, the curves converge as they become more representative of a distributed element. It is interesting to note that the shapes of higher order curves enter all quadrants of the Nyquist diagram at high frequency, unlike those commonly seen for electroceramics impedance spectra (such as those in [3]). This can be explained by the frequency-domain flux-to-temperature transfer function, Z_{th}, of the two-port Cauer network, which, for an n^{th} order network, takes the form below. This equation can be derived using the leapfrog ladder technique noting that the input is a current source [7].

$$Z_{\text{th}}(j\omega) = \frac{1}{A_n(j\omega)^n + A_{n-1}(j\omega)^{n-1} + \ldots + A_1(j\omega) + A_0} \qquad (3)$$

where $\mathbf{A}$ is a set of constants. Since each of $(j\omega)^1$ to $(j\omega)^4$ occupy a different quadrant in the Nyquist plot, a curve in all quadrants can be approximated using a Cauer network of $n \geq 4$. The $n = 1$ curve is especially inaccurate because it represents a first order network and therefore occupies only one quadrant. It shows significant difference from the distributed element over all frequencies. This demonstrates the limitations inherent in modelling a single material block or device using a single element network.

Measurement position: For a high-order Cauer network, the position in the network at which the temperature is measured also has an impact. A two-port Cauer network with several measurement points is shown in Fig. 4. By plotting Nyquist curves of the transfer impedance between the heat flux source, Q, and each temperature, T, the effect of several adjoined $R_{x(\text{th})}C_{x(\text{th})}$ elements can be analysed, as shown in Fig. 5.

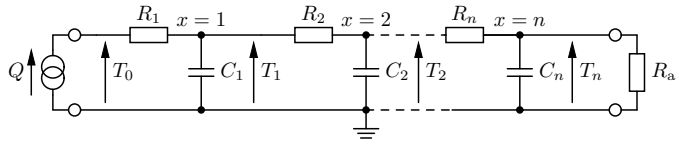

Fig. 4 *Cauer two-port network with temperature measurements taken at various points along the material.*

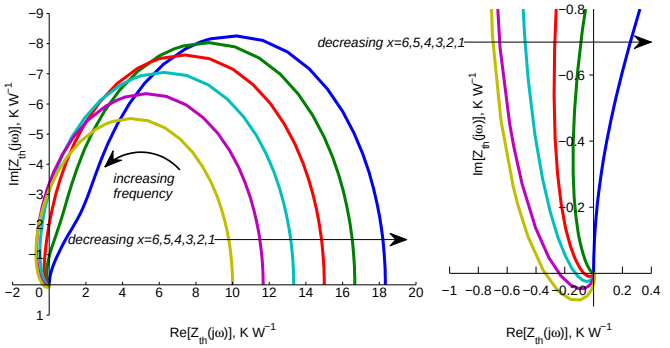

Fig. 5 *Nyquist plots of single material Cauer network approximation with transfer impedance taken at different nodes, x*

At the heat source, the curve is first order and therefore occupies only a single quadrant. As the measurement is taken further away from the heat source, the curves take a form similar to the distributed element seen in Fig. 3. With each additional element between source and measurement, the curves occupy an extra quadrant of the Nyquist diagram. Therefore for an accurate approximation of the transfer impedance part-way through a material, at least four Cauer elements are required between heat flux source and temperature measurement.

We have seen that it is important to model a thermal circuit with sufficient order to capture behaviour in all quadrants, particularly in situations where the high frequency response is important. One such situation is the thermal management of power devices under rapidly changing load, for example in high pulsed power applications, where the resulting large changes in temperature cause wire bond failure and shorten device lifespan [1].

In this case, the first order model does not represent a good approximation of a distributed element at high frequency although it shows better agreement at low frequency. This is because, unlike the distributed element result, it cannot cross the imaginary axis and therefore represents an inaccurate model for applications with rapid power change.

Conclusion: Thermal impedance spectroscopy is used to assess the required number of Cauer network elements to model a single material block. Although such blocks are commonly modelled as first order Cauer networks, their thermal transfer impedance occupies only a single quadrant of a Nyquist diagram in contrast to experimental measurements. A Cauer network consisting of at least four elements between the heat flux input and the temperature measurement is therefore required for the model to correctly occupy all quadrants of the Nyquist diagram at high frequency. Results demonstrate that Cauer networks with orders higher than four does not substantially improve the approximation to distributed components, indicating that four elements is a sensible compromise between accuracy and complexity.

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