Self-Organised Criticality

Non-linear systems

Huw Birch

The University Of Sheffield.

E-Futures

Outline

- Aims
- Van der Pol oscillator
- Unforced - Natural oscillation
- Forced:
  - Periodic forcing
  - Random forcing
- Further work

Aims

- Want to model a sustainable energy system using self-organised criticality and entropy
- Use Van der Pol model
- Link results to entropy

Van der Pol oscillator

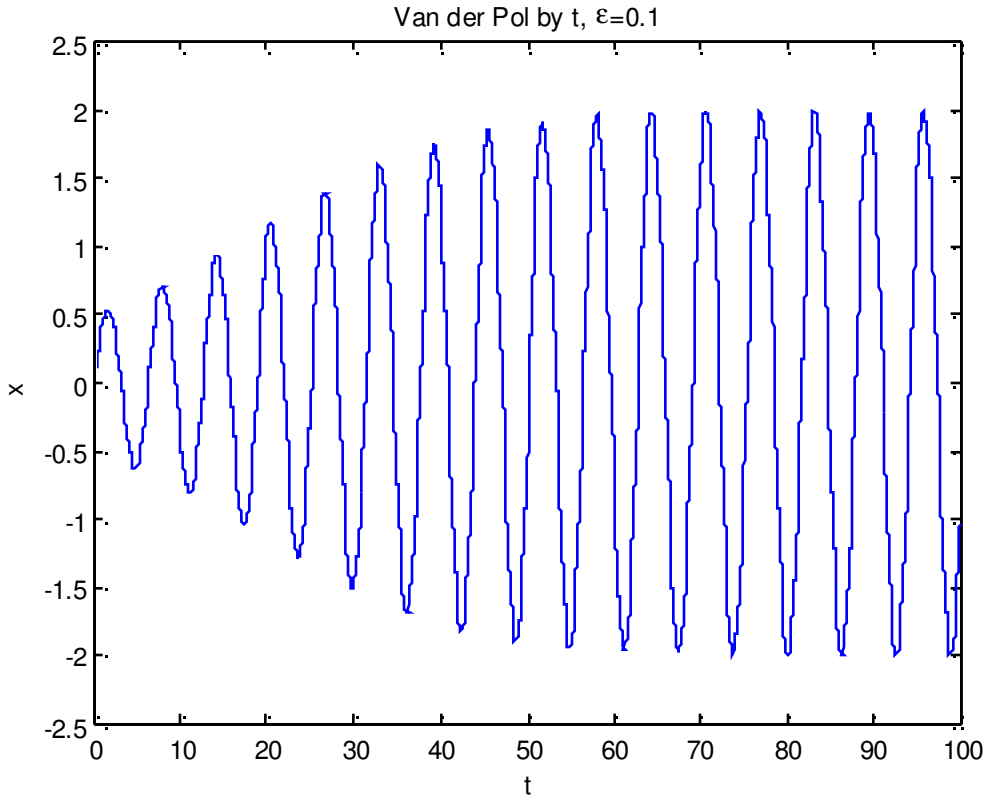
- Typical model for self-organisation (applicable to many systems)
- Based on model of an oscillating particle
- Nonlinear differential equation

$$\ddot{x} - \varepsilon(1 - x^2)\dot{x} + x = 0$$

- Constant ε is a *control parameter* – governs damping force

Natural Oscillation

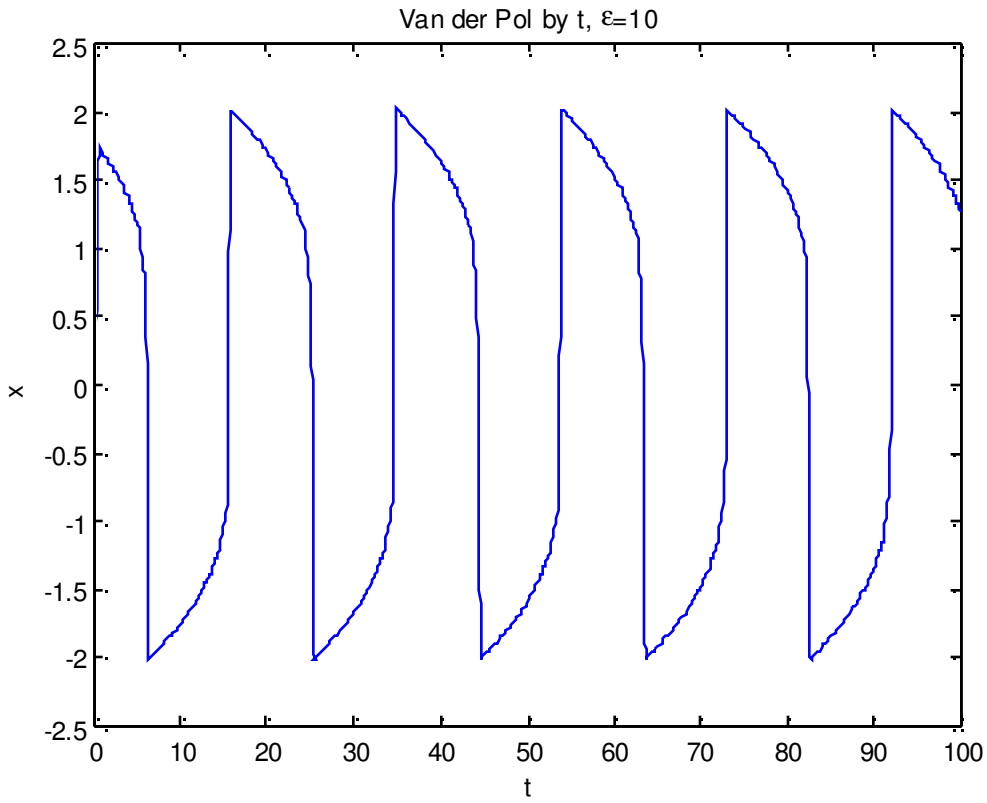
Low damping force (ε=0.1)



Falls into steady state (limit cycle) with max deviation 2 from origin

Natural Oscillation

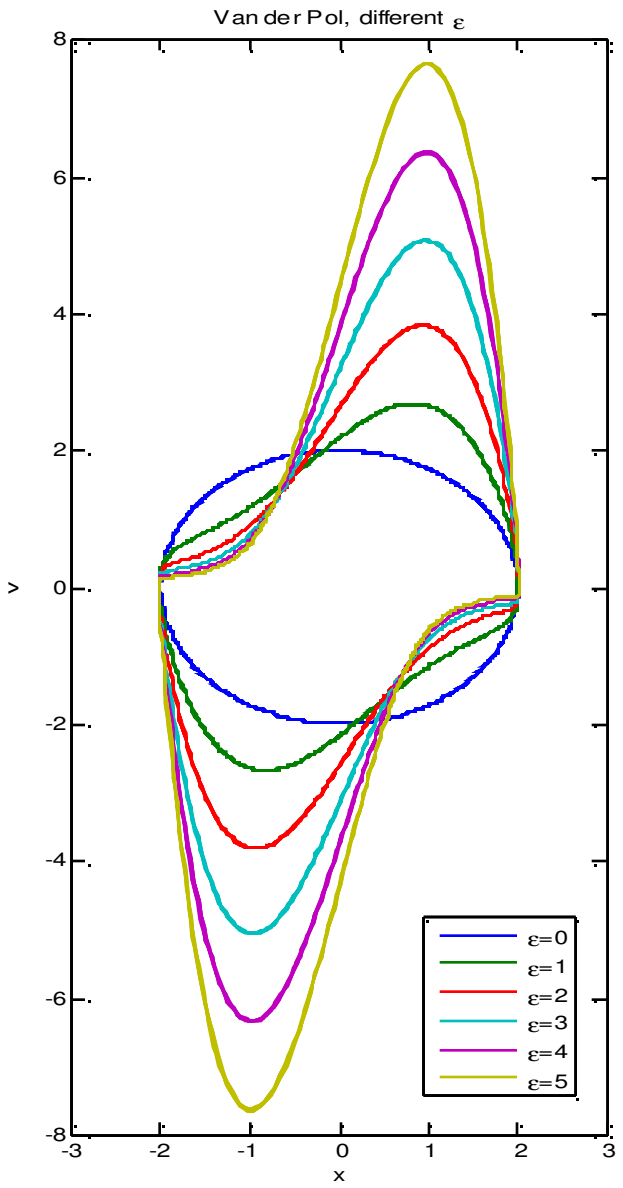
High damping force (ε=10)



Same deviation from origin, but longer period of oscillation and sharper change in direction at max distance

Limit cycles

- The repeating oscillation that the system falls into is called its *limit cycle*, or attractor
- The movement around the limit cycle is clockwise
- Limit cycle becomes more distorted as ε increases



Periodic forcing

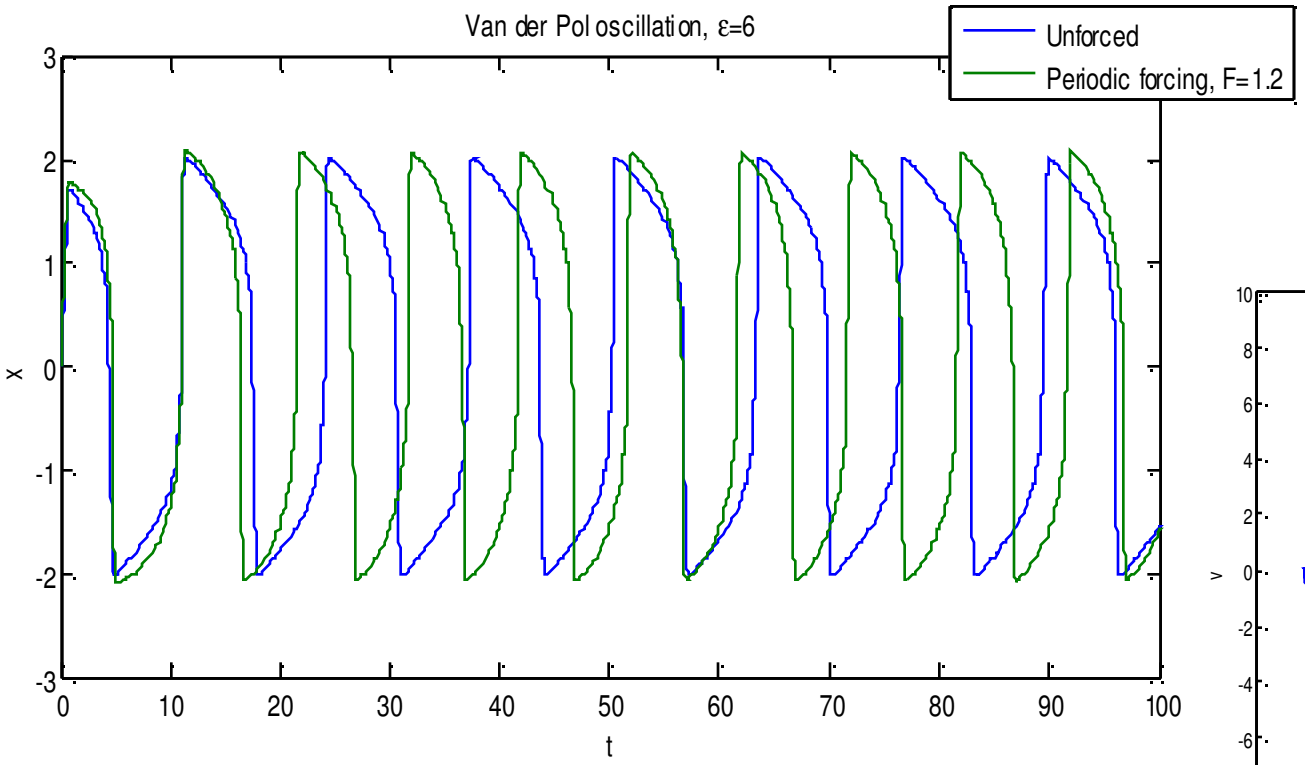
- Limit cycle changes due to external oscillating force:

$$\ddot{x} - \varepsilon(1 - x^2)\dot{x} + x = F cos\left(\frac{2\pi t}{T_{in}}\right)$$

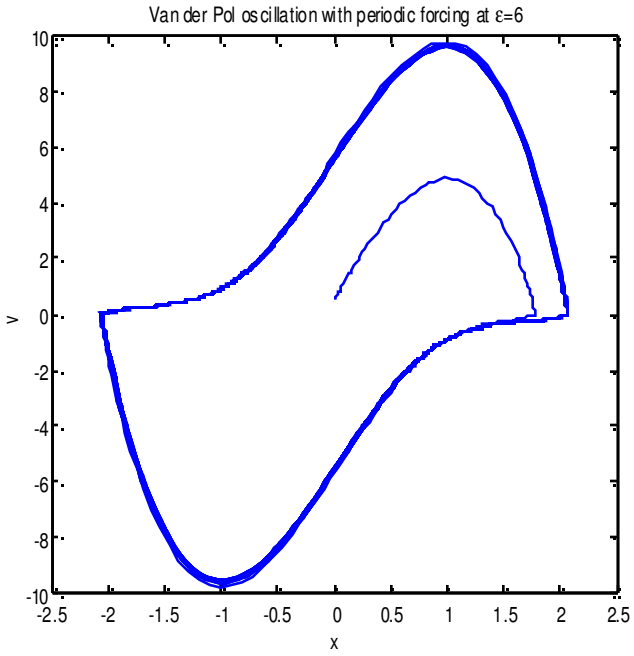
- Results depend on the value of ε, F, and T_{in}
- Chaotic or periodic

Modelling forcing

Periodic result (ε=6)

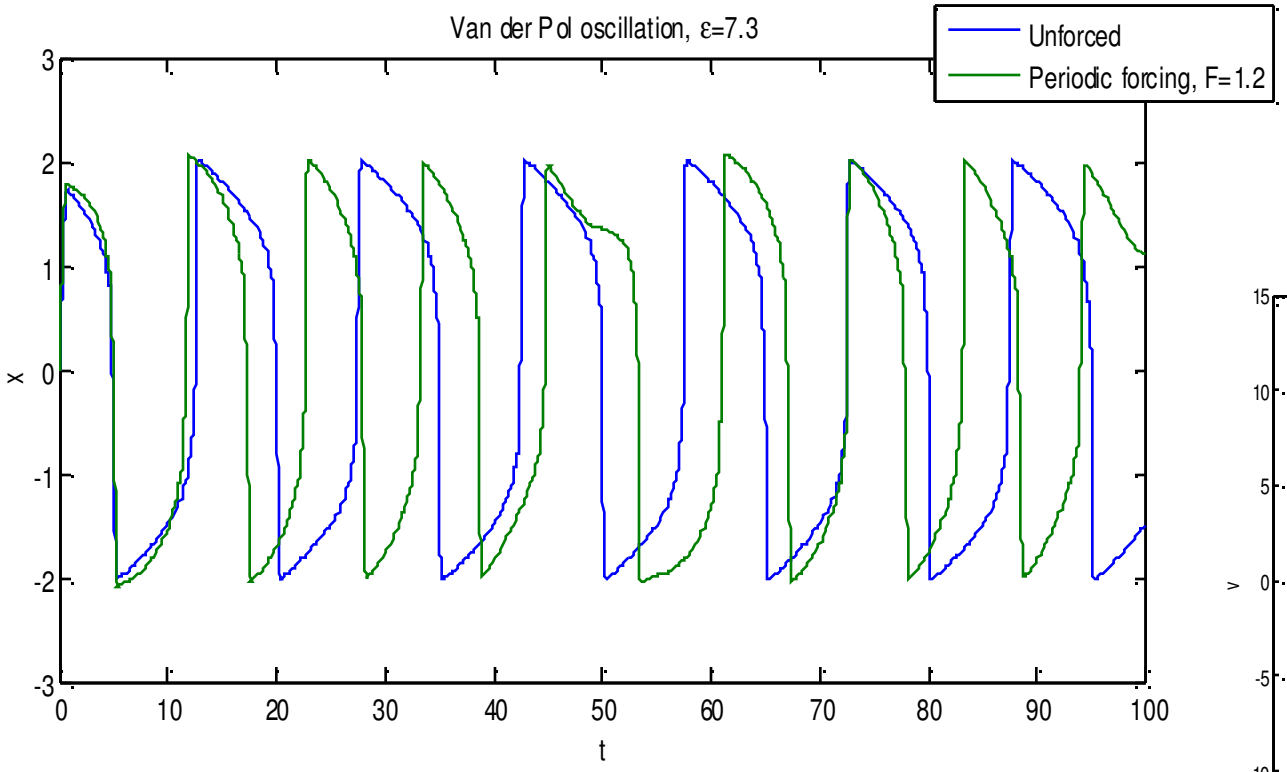


Falls into a limit cycle different to previous, but still stable looking

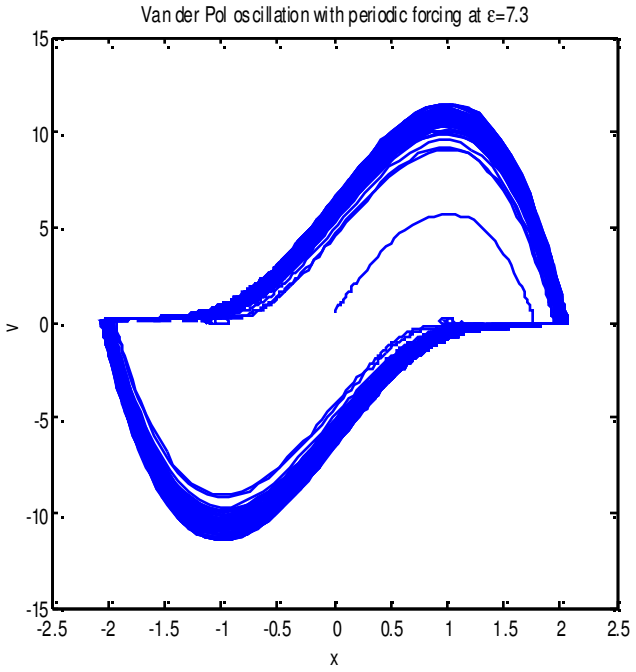


Modelling forcing

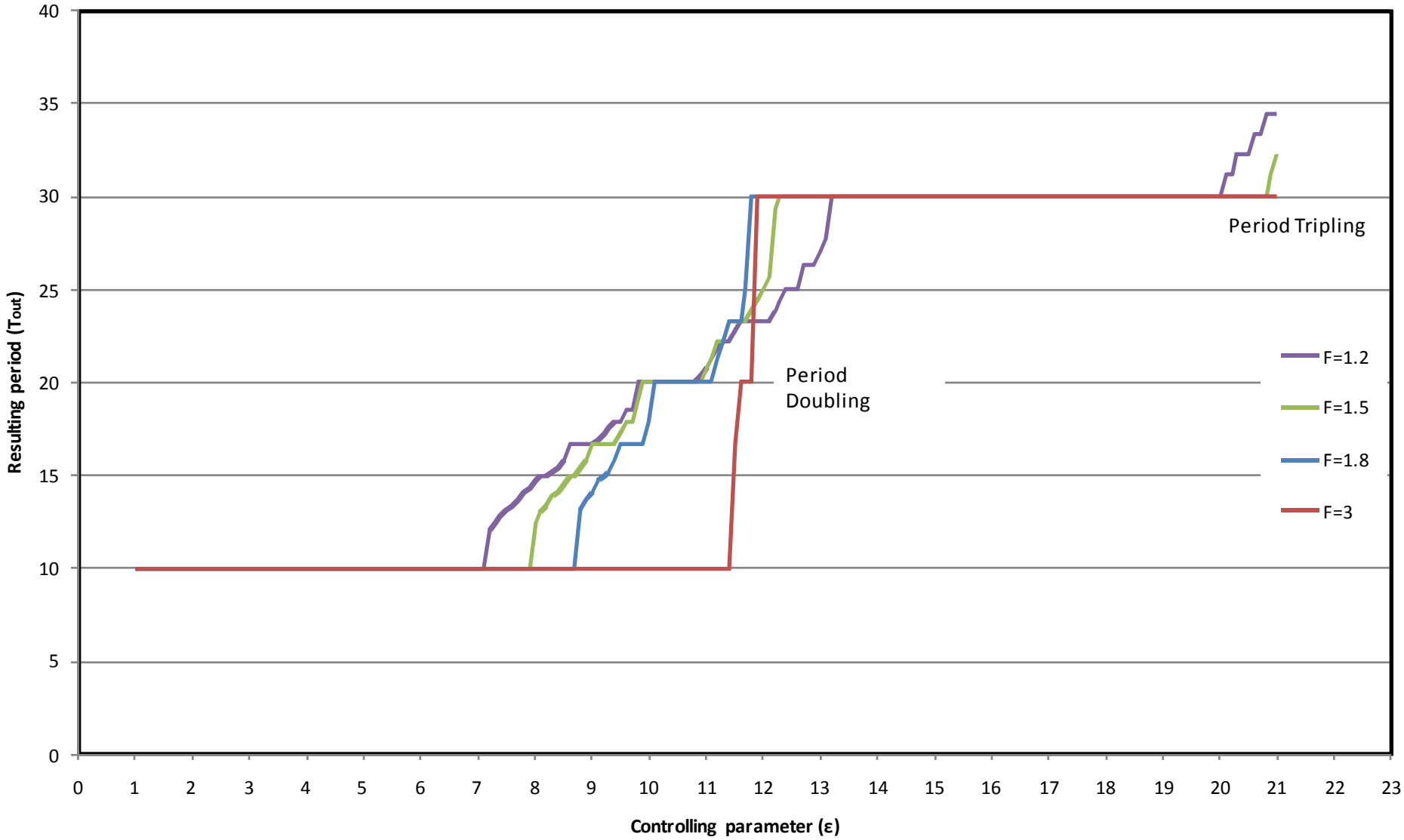
Chaotic result (ε=7.3)



No proper limit cycle reached



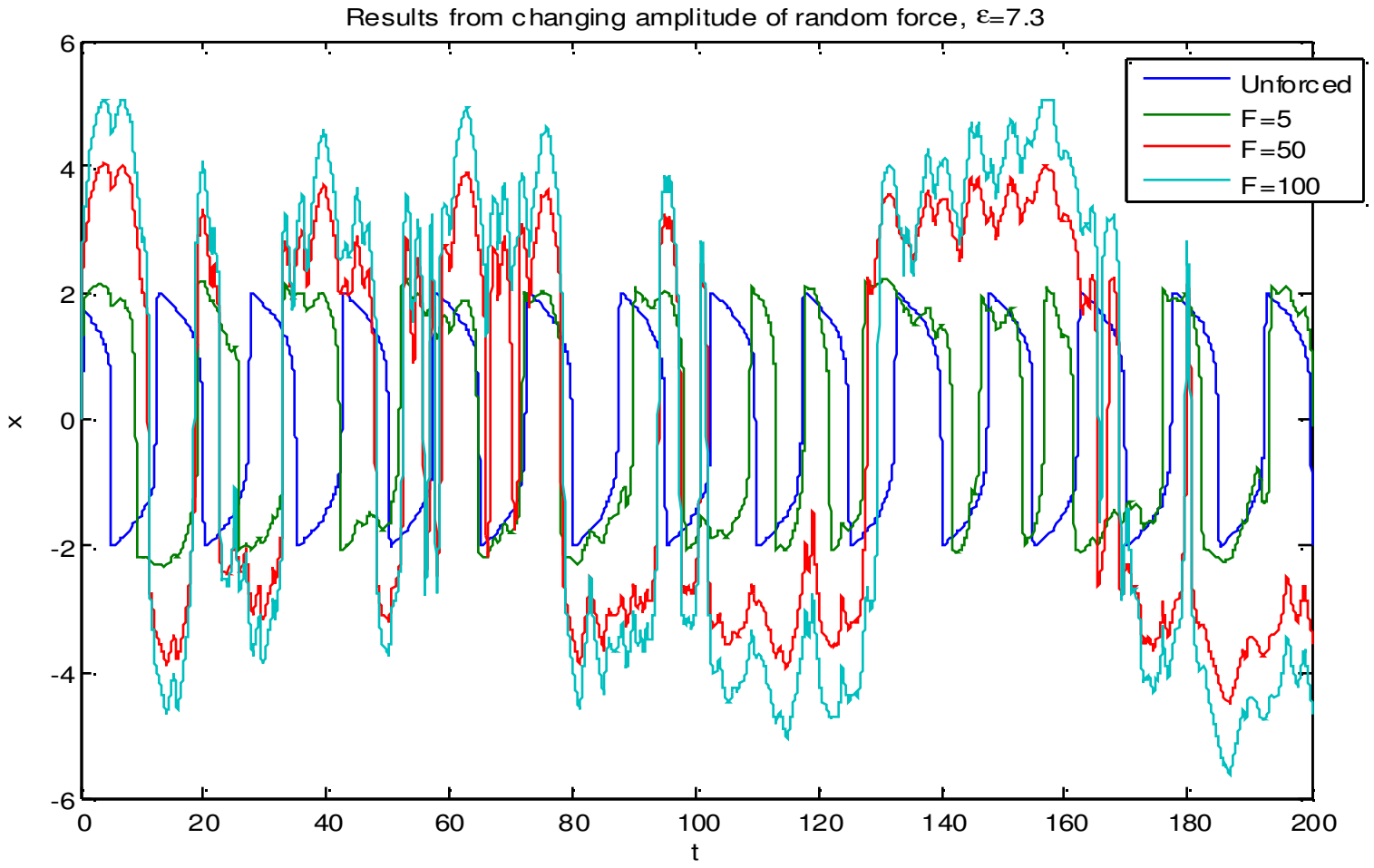
Chaotic regions



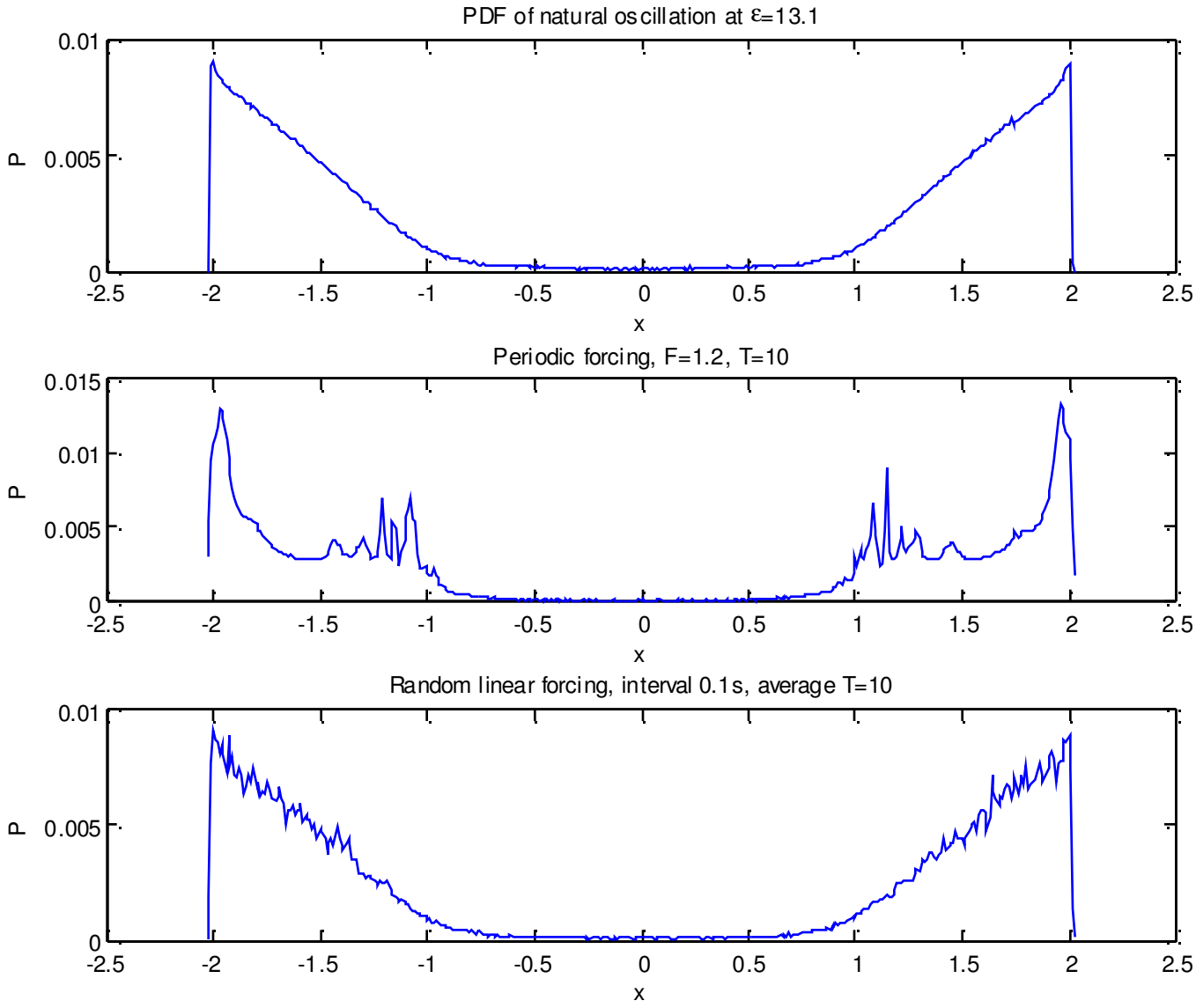
Random forcing

- The period T_{in} of the external force was randomised between 0 and 20
- Randomly changes every second (correlation time=1)
- Model unknown stochastic forces, or noise

Random forcing



Probability Distributions



Entropy

- Measures missing information
- Powerful concept for understanding strongly nonlinear system
- Need for generalised entropy to understand strongly nonlinear system (esp. self-organisation)
- Aim in future is to link generalised entropy to self-organisation

References

- Chaos in Dynamical Systems, Edward Ott.
- Self-Organization in the van der Pol Generator for Nonextensive Systems, R. G. Zaripov. Theoretical and Mathematical Systems.
- Criteria of Self-Organization, Yu L. Klimonovich. Chaos, Solitons and Fractals Vol. 5.
- Period-doubling cascades and devil's staircase of the driven van der Pol oscillator, Ulrich Parlitz & Werner Lauterborn. Physical Review A.
- Stochastic transient of a noisy van der Pol oscillator, H. K. Leung. Physica A.
- Noise-induced critical breakdown of phase lockings in a forced van der Pol oscillator, Shinji Doi. Physics Letters A.
- Stochastic bifurcations and coherencelike resonance in a self-sustained bistable noisy oscillator, A. Zakharova. Physical Review E.

Thank you for listening.

Any questions?