PARAMETERISATION OF LEAD ACID BATTERIES

MINI-PROJECT REPORT

THREE PAGE PRÉCIS

Jonathan Davidson
Jonathan.Davidson@sheffield.ac.uk

MAY 2011
PARAMETERISATION OF LEAD ACID BATTERIES

Mini project report three page précis
Jonathan Davidson (Jonathan.Davidson@sheffield.ac.uk)

INTRODUCTION

In today’s world, the battery plays a vital role in local energy storage for a range of systems. It is important to be able to determine the state of charge (that is, how much energy is available from the battery in its present state) and the state of health (how much energy is available at full charge). Commonly, these parameters are calculated by calculating how much charge has left a battery of known capacity (coulomb counting) and adjusting the full capacity every time a full discharge is observed (with instances of full charge and no charge recognised by the characteristic voltages observed in these states).

Electric vehicles pose a difficulty for this system: it is very unusual for batteries to be fully discharged. Coulomb counting techniques require an occasional full discharge to calibrate the system. The consequences of this difficulty are compounded by occasional full discharge to calibrate the system. The discharged. Coulomb counting techniques require an accurate state of charge information: a car which indicates it has charge enough for 50 miles but runs out after 30 is a problem when making a 35 mile journey. Whereas for liquid fuel vehicles, the size of the tank is fixed and how much fuel remains is easy to measure, for electric vehicles special techniques are required.

Battery cells are complicated electrochemical systems. If we can use non-intrusive techniques to infer the internal structure and oxidation state of the cell, we will be better placed to determine state of charge and health which are heavily influenced by these factors.

In this report we study a method to determine the parameters associated with Randle’s model of electrochemical cells, an electrical equivalent circuit of a cell, since these parameters are influenced by the structure and are known to be affected by state of charge and health.

Lead acid battery

A great deal of literature has been published on the calculations of parameters for Randle’s model. Two versions of the model are currently used and shown in figure 1. The first is Randle’s original model which includes the Warburg element to represent charge transfer across the electrolyte. A simplified model assumes this is a bulk capacitance (in which the battery’s energy is stored) and splits it from the charge transfer resistance to simplify the mathematics [1-2].

Figure 1: Randle’s model (a); simplified (b)

The parameters represent various structural features of the cell which can be observed by recording the frequency-dependant impedance. $R_l$ represents the lumped series resistance. $C_s$ represents the surface capacitance due to the double layer between electrolyte and electrodes [3-4]. $R_t$ refers to the charge transfer resistance: that is, the losses due to ionisation of lead. $C_b$ and the Warburg element $W$ refer to the bulk storage of charge. In the simplified model, a very large capacitance (many thousands of coulombs at least for a car battery) suffices for this. Other reports consider the Warburg element as an alternative; however, this adds unnecessary complexity [4].

Pseudo-random binary sequences

In an ideal world, the impedance of a battery at a certain frequency would be measured by applying a sine wave current at the frequency and measuring the corresponding voltage amplitude and phase, then repeating over all interesting frequencies—this is impedance spectroscopy. A faster and simpler approach is to subject the battery to pseudo-random binary sequences (PRBS) which contain all frequencies up to a limit.

PRBS is a sequence of 1s and 0s where the next value of the sequence is hard to predict. It is generated using linear shift feedback registers. A 4 bit sequence (leading to 15 ‘1’s and ‘0’s) is realised by this circuit. (Taps from [5].)

By comparing the frequency responses of the idealised system with the actual systems, the parameters of the model can be set to give the two the greatest match and hence record the parameters.
Implementation

The impedance spectrum is required to parameterise Randle’s model and was obtained using a 15 bit 10 Hz 10 A PRBS discharge of a battery with 80 Hz synchronised measurement of terminal current and voltage for the following reasons:

- 0 and 10 A discharge for PRBS ‘0’ and ‘1’ causes sufficient voltage perturbations to record while the average current (5 A) is low enough to not affect state of charge greatly.
- 80 Hz synchronised measurement means 40 Hz and lower is useful and unaliased.
- 10 Hz signalling means that the -3 dB point is 2.5 Hz. Frequencies above are more prone to noise and there are frequent singularities (see figure 2) but the data at these frequencies remains useful.
- 15 bit PRBS creates a 32,767 length sequence therefore valid to 0.3 mHz.

The power spectrum of a 15 bit PRBS is shown in figure 2. The spectrum is identical to white noise at frequencies below around \( f_{\text{PRBS}} \). Above this, there are regular singularities so that above around \( 5 \times f_{\text{PRBS}} \), the data are unusable.

![Figure 2: Spectrum of 15 bit PRBS with -3 dB point identified. Smoothed with unsmoothed inset](image)

The impedance spectrum is the quotient of the measured voltage spectrum and current spectrum.

Current-controlled discharge design

In the first instance, PRBS generation and measurement were performed using a LabView-controlled variable current source and measurement rig, shown below.

![LabView Output](image)

PRBS is performed by generating the appropriate sequence in software, and producing it as 10 A switched current demand. Voltage and current are measured at suitable intervals and data is analysed using MATLAB. This method still requires large equipment: a cheaper, simpler and smaller method would be favourable.

Since over the course of a discharge, the d.c. battery voltage doesn’t change much, discharging through a resistor instead of using a current source should not cause significant error. A simple PIC-controlled circuit was constructed with this topology.

![Simple Circuit](image)

The current is calculated from the voltage drop across the resistor \( (V_{\text{batt}} - V_{\text{sw}}) \) and the battery voltage is measured directly. The circuit is designed for high accuracy and a measured range targeted to expected results.

This method has significant advantages over the use of a rig. Firstly, it is far smaller and more simple, making it much cheaper to design and integrate to a battery system. It has minor problems: the resistor has slight temperature dependence and parasitic inductance. Avoiding transients and temperature changes is easy and solves this.

Data read by the PIC is transferred to a computer by RS232, and processed in MATLAB. A comparison between results from the rig and this switched resistance method shows no difference in results except a reducing in noise for this design due to the high sensitivity and targeted voltage range of the device.

Results and discussion

Once data has been acquired, the simplified Randle’s model is fitted to it and the parameters recorded using this method: the parameters are initially estimated using a method described by Fairweather et al [1]. A battery is subjected to a long constant current discharge which is interrupted for about a second. \( R_t, R_i \) and \( C_s \) affect readings in this time frame and are calculated from transient effects as shown below.

![Results](image)
Parameterisation of Lead Acid Batteries

May 2011  Jonathan Davidson

$C_b$ is calculated from the long term discharge rate. Analysis yields $R_t = 11.6 \text{ m}\Omega$; $R_i = 6.0 \text{ m}\Omega$; $C_a = 14 \text{ F}$.

These estimations are refined using the frequency domain from PRBS testing, which has the advantage of taking a large data set into account. The transfer function of the simplified Randle’s model is

$$Z(s) = \frac{C_b C_e R_i R_t s^2 + (C_p R_t + C_b R_t + C_e R_i) s + 1}{C_j R_t C_b s^2 + C_b s}$$

The same battery was subjected to a 10 A 15 bit 10 Hz PRBS and its frequency response recorded. This is compared to the ideal response for the parameter values above: (full plot top; zoomed bottom)

![Graph showing frequency response](image)

The initial estimate of the model and its parameters is good since the shape and values of both model and actual spectra match. The shape is characteristic of Randle’s model and has a flat region at high frequency, a slight gradient as frequency decreases followed by a large ramp at low frequency. The parameters are refined by altering the model’s parameters until the model and actual spectra match. For instance, our initial estimate of $R_t$ is 11.6 mΩ. We can compare the battery spectrum to spectra with $R_t$ of 5 to 50 mΩ, as we do in figure 3. From these spectra, we can see that $R_t = 14.0 \text{ m}\Omega$ is the closest match (we look at the higher frequencies since series resistance dominates here). The values of other parameters are left unchanged.

![Figure 3: Actual spectrum compared to model spectra at various $R_t$s](image)

This process is repeated for each parameter and the best fit is taken. $R_t$ and $C_b$ are most easily refined using smoothed plots since they affect the noisiest area of the spectrum. Taking this approach, the results are: $R_t = 14 \text{ m}\Omega$; $R_i = 2.5 \text{ m}\Omega$; $C_a = 70 \text{ F}$; $C_b = 65 \text{ kF}$.

The spectra match well now, with the only significant error occurring at the start of the very low frequency ramp where the model underestimates the impedance. This error is due to inaccuracies in the simple model used.

The model does, however, provide a good estimation in the time domain, as we can see below.

![Graph showing time domain](image)

**Parameters for different batteries**

Having developed a method for calculating the parameters, a special battery for electric cars consisting of five 6-cell lead acid batteries in parallel with a lead acid supercapacitor was analysed. These batteries were separated and tested individually, but also tested as a single unit. It was that the single unit matched models predicted from each constituent battery in parallel and all elements from constituent batteries lumped together. This despite the battery having been observed to have unusual time domain responses.

**Conclusion**

From existing literature it was known that the complex impedance spectrum of batteries is determined by their physical structure, state of charge and state of health [6]. Randle’s model has been parameterised using PRBS-generated frequency responses.

**Further Work**

Verification of results using complex impedance spectroscopy is the next step. This method gives more reliable data and the phase information it yields allows the battery model to be refined [7]. To determine the relationship between state of health and Randle’s model, a battery can be taken and cycled with repeated parameterisation every few cycles.

**References**


